Real functions

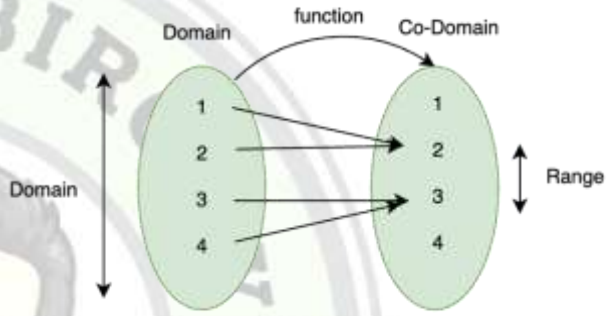
Real function :

The function f : X → Y is called a real function

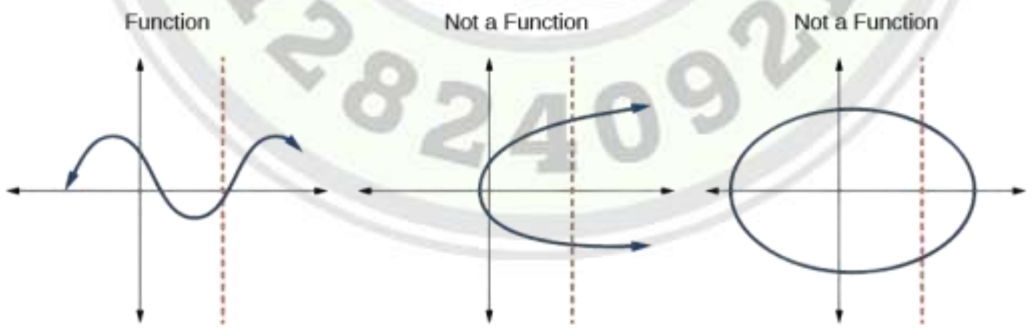
if each of the domain (X) and the co-domain (Y) is The set of real number

or a proper subset of it

- the relation is a function if every value of the variable x ∈ X is related with only one value of variable y ∈ Y


Domain
function
Co-Domain
Domain
1
2
3
4
1
2
3
4
Range

- the relation is not a function if there exists at least one straight line parallel to y–axis and intersects the graph of the relation at more than one point


Function
Not a Function
Not a Function

---

the domain of the Real function

● Polynomial Function :

The domain is R or a subset of it .

EX : ■ f (x) = 4 → (constant function)

■ f (x) = 2 x – 3 → (linear function)

■ f (x) = $x^2 - 4$ → (quadratic function)

Notes :

● f : f (x) = $\frac{x-3}{2}$ a Polynomial Function

becouse we can put it at a form : f (x) = $\frac{1}{2}x - \frac{3}{2}$

then its domain is R

● Rational Function :

The domain is R - { zeroes of the denominator }

◉ state the domain :

⟹ f (x) = $\frac{x-4}{3x}$

SOL

………

⟹ f (x) = $\frac{x-2}{2x+4}$

SOL

………

⟹ f (x) = $\frac{3}{\frac{1}{2}x}$ in R = ……………………

SOL

………

⟹ f (x) = $\frac{5}{x^2 - 4x - 5}$

SOL

⟹ f (x) = $\frac{2\,x - 4}{x^2 + 16}$

SOL

⟹ f (x) = $\frac{2\,x - 4}{x^2 + 16}$

SOL

⟹ f (x) = $\frac{x}{\sqrt[3]{x}}$

SOL

● the n^{th} root Function :

if : ƒ (x) = $\sqrt[n]{h\ (x)}$ where : n ∈ z^+ , n $>$ 1

First : when (n) is an odd number

▶ the domain = R

Second : when (n) is an even number

▶ the domain is the set of all values of x which satisfy h (x) ≥ 0

◉ state the domain :

⟹ ƒ (x) = $\sqrt{2\ x-6}$

SOL

....

....

⟹ ƒ (x) = $\sqrt[3]{x^2-9}$

SOL

....

....

⟹ f (x) = $\sqrt{4+4\,x+\ x^2}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\sqrt{x^2-3\,x-4}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\sqrt{9-\ x^2}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

$\Longrightarrow$ f (x) = $\sqrt{4\,x^2-25}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

$\Longrightarrow$ f (x) = $\dfrac{3\,x-1}{\sqrt{x^2-4\,x}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

$\Longrightarrow$ f (x) = $\dfrac{3\,x-1}{x\,.\sqrt[3]{x^2+x-2}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ $f(x) = \dfrac{2x - 6}{x\sqrt{x - 3}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ $f(x) = \dfrac{2x - 4}{\sqrt{x - 4} - 4}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ $f(x) = \dfrac{x}{\sqrt{x - 2} + 1}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\frac{\sqrt{x-1}}{\sqrt{2x-6}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\sqrt{\frac{x-1}{2x-6}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\frac{\sqrt{3-x}}{\sqrt{4-2x}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\sqrt{\frac{3-x}{4-2x}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\frac{\sqrt{x^2-4}}{\sqrt{x^2-4x+3}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ f (x) = $\sqrt{\frac{x^2-4}{x^2-4x+3}}$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

$\Longrightarrow$ f (x) = $\dfrac{\sqrt{x^2 - 1}}{\sqrt{6 + 5\,x - x^2}}$

SOL

……

……

$\Longrightarrow$ f (x) = $\sqrt{\dfrac{x^2 - 1}{6 + 5\,x - x^2}}$

SOL

……

……

$\Longrightarrow$ f (x) = $\dfrac{1}{\sqrt{6\,x - 9 - x^2}}$

SOL

……

……

● the piecewise Function :

it is the function that defined by different rules

For different parts of its domain .

◉ Determine the domain :

$$\Longrightarrow f(x) = \begin{cases} 3 - x & , \quad x < 0 \\ x - 3 & , \quad x > 0 \end{cases}$$

SOL

....

....

$$\Longrightarrow f(x) = \begin{cases} x^2 - 3 & , \quad x < 2 \\ x + 2 & , \quad 2 < x < 6 \end{cases}$$

SOL

....

....

◉ Multiple choice questions :

⟹ in all following relations , y is a function in x except ………

(a) $y + 1 = 2x$ (b) $\frac{1}{2}y = x^2 - 2$ (c) $2x = y^2 + 1$ (d) $y = \tan x$

SOL

⟹ in all following relations , y is a function in x except ………

(a) $y = -\cos x$ (b) $y = -2$ (c) $2y = x^2 + 1$ (d) $y^2 = x^2 + \frac{1}{2}$

SOL

⟹ the domain of the function ƒ : ƒ (x) = 3 is ………

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{3\}$ (d) $\{0, 3\}$

SOL

$\Longrightarrow$ the domain of the function ƒ : ƒ (x) = $\frac{2x+1}{x-2}$ is ………

(a) R (b) R – { - $\frac{1}{2}$ } (c) R - { - $\frac{1}{2}$, 2 } (d) R – {2}

SOL

$\Longrightarrow$ the domain of the function ƒ : ƒ (x) = $\frac{x+2}{(x+2)(x-2)}$ is ………

(a) R (b) { 2 , - 2 } (c) R – { 2 } (d) R – {2 , - 2}

SOL

$\Longrightarrow$ the domain of the function ƒ : ƒ (x) = $\frac{x^2+1}{x^2+3x}$ is ………

(a) R – {1 , - 1} (b) R – { 0 , - 3 } (c) R (d) R – {0 , 3}

SOL

⟹ the domain of the function ƒ : ƒ (x) = $\frac{2x-6}{x^2+x+1}$ is ………

(a) R (b) R – { 6 } (c) R – { 3 } (d) R – { 1 }

SOL

………

⟹ the domain of the function ƒ : ƒ (x) = $\frac{2}{x^3-x}$ is ………

(a) R – { 3 } (b) R – { 2 } (c) R – { 0 , 1 } (d) R – { 0 , 1 , -1}

SOL

………

⟹ the domain of the function ƒ : $R^+ \rightarrow R$, ƒ (x) = $\frac{x-1}{4x}$ is ………

(a) R (b) R – {0} (c) R^+ (d) R – {1}

SOL

………

⟹ if the domain of the function ƒ : ƒ (x) = $\frac{2}{x^2 - 6\,x + k}$ is R – {3} , then : k = ...

(a) 3 (b) 9 (c) ± 9 (d) 18

SOL

....

⟹ the domain of the function ƒ : ƒ (x) = $\sqrt{4 - x}$ is ....

(a) [4 , ∞ [(b)] - ∞ , 4 [(c)] 4 , ∞ [(d)] - ∞ , 4]

SOL

....

⟹ the domain of the function ƒ : ƒ (x) = $\sqrt{x - 3}$ is ....

(a) R (b) R - { 3 } (c) [3 , ∞ [(d)] - ∞ , 3 [

SOL

....

⟹ the domain of the function ƒ : ƒ (x) = $\sqrt[3]{x-5}$ is ……………

(a) [5 , ∞ [(b)] - ∞ , 5 [(c) R (d) R^+

SOL

……………………………………………………………………………………

⟹ the domain of the function ƒ : ƒ (x) = $\sqrt[3]{9-x^2}$ is ……………

(a)] - 3 , 3 [(b) R (c) R -] - 3 , 3 [(d) [- 3 , 3]

SOL

……………………………………………………………………………………

⟹ the domain of the function ƒ : ƒ (x) = $\sqrt{x+3}$ is ……………

(a)] - 3 , ∞ [(b) [- 3 , ∞ [(c) {- 3} (d) {0}

SOL

……………………………………………………………………………………

⟹ the domain of the function ƒ : ƒ (x) = $\sqrt{x^2 + 9}$ is ……………

(a) R (b) R – { - 3 , 3 } (c) R – [- 3 , 3] (d)] - 3 , 3 [

SOL

……

⟹ the domain of the function ƒ : ƒ (x) = $\frac{5}{\sqrt{x - 4}}$ is ……………

(a) [4 , ∞ [(b)] 4 , ∞ [(c)] - ∞ , 4] (d)] - ∞ , 4 [

SOL

……

⟹ the domain of the function ƒ : ƒ (x) = $\frac{1}{\sqrt{9 - x^2}}$ is ……………

(a) R (b) R - [- 3 , 3] (c) R - {- 3 , 3 } (d)] - 3 , 3 [

SOL

……

⟹ **the domain of the function f : f (x) = $\sqrt{9-x}$ is ……………**

(a) R **(b) R - {9}** **(c)] - ∞ , 9]** **(d) [9 , ∞ [**

SOL

……………………………………………………………………………………………………

⟹ **the domain of the function f : f (x) = $\sqrt[4]{x^2+4}$ is ……………**

(a) R **(b) R – { 4 }** **(c) R – { 0 }** **(d) R – { - 2 , 2 }**

SOL

……………………………………………………………………………………………………

⟹ **the domain of the function f : f (x) = $\sqrt{x^2-25}$ is ……………**

(a) R – [- 5 , 5] **(b) R –] - 5 , 5 [** **(c)] - 5 , 5 [** **(d) [- 5 , 5]**

SOL

……………………………………………………………………………………………………

⟹ the domain of the function f : f (x) = $\frac{1}{|x| - 3}$ is ……………

(a) { - 3 , 3 } (b) [- 3 , 3] (c) R - [- 3 , 3] (d) R - { - 3 , 3 }

SOL

……

⟹ if the domain of the function f : f (x) = $\sqrt{a - x^2}$ is [-3 , 3] , then : a = ………

(a) 3 (b) 9 (c) ± 3 (d) ± 9

SOL

……

⟹ if the domain of the function f : f (x) = $\sqrt{b - a\,x^2}$ is R -] -2 , 2 [

then : a b = ……

(a) ± 4 (b) - 4 (c) 8 (d) 16

SOL

……

⟹ if the domain of the function f : f (x) = $\frac{2}{\sqrt{x^2+a}}$ is R – [-5 , 5]

then : a = …………

(a) 5 (b) -25 (c) - 5 (d) 25

SOL

………

⟹ if the domain of the function f : f (x) = $\frac{x-1}{\sqrt{x^2+a}}$ is R

then : a can not to be = ……………………

(a) 5 (b) $\sqrt{4}$ (c) 0 (d) 9

SOL

………

⟹ if the domain of the function f : f (x) = $\frac{1}{\sqrt{x-a}}$ is] -3 , ∞ [, then : a = ………

(a) 3 (b) -3 (c) ± 3 (d) $\sqrt{3}$

SOL

………

⟹ the domain of the function f : f (x) = $\frac{3x-4}{\sqrt{x+6}-2}$ is ………

(a)]-6 , ∞ [(b) [-6 , ∞ [– {4} (c) [-6 , ∞ [(d) [-6 , ∞ [– {-2}

SOL

……………………………………………………………………………………

⟹ if the domain of the function f : f (x) = $\frac{x-2}{\sqrt{x-a}-b}$ is [3 , ∞ [– {4}

then : a + b = ………

(a) 1 (b) -2 (c) 4 (d) 7

SOL

……………………………………………………………………………………

⟹ the domain of the function f : f (x) = $\frac{5}{\sqrt{x-1}-3}$ is ……………

(a) [1 , ∞ [(b) [1 , ∞ [- {3} (c) [1 , ∞ [- {10} (d) [-3 , ∞ [

SOL

……………………………………………………………………………………

$\Longrightarrow$ **the domain of the function f : f (x) = $\frac{7}{\sqrt{x-1}+3}$ is ……………**

(a) [1 , ∞ [(b) [1 , ∞ [- {-3} (c) [1 , ∞ [- {10} (d) [3 , ∞ [

SOL

……………………………………………………………………………………………………

$\Longrightarrow$ **the domain of the function f : f (x) = $\frac{x}{\sqrt{x-1}-x}$ is ……………**

(a)] 0 , ∞ [(b)] - ∞ , 0 [(c) [0 , ∞ [– {1} (d)] 0 , ∞ [– {3}

SOL

……………………………………………………………………………………………………

$\Longrightarrow$ **the domain of the function f : f (x) = $\sqrt{6x-9-x^2}$ is ……………**

(a) R (b) R - {3} (c) [3 , ∞ [(d) {3}

SOL

……………………………………………………………………………………………………

⟹ the domain of the function f : f (x) = $\sqrt{\dfrac{x-5}{x+4}}$ is ………

(a) [5 , ∞ [(b) [- 4 , 5] (c) R - [- 4 , 5 [(d) R -] - 4 , 5]

SOL

⟹ the domain of the function f : f (x) = $\sqrt{\dfrac{3-x}{5-x}}$ is ………

(a) R -] 3 , 5] (b) R - { 3 , 5 } (c) [3 , 5 [(d) R - [3 , 5 [

SOL

⟹ the domain of the function f : f (x) = $\dfrac{\sqrt{x-3}}{\sqrt{5-x}}$ is ……

(a)] 3 , 5 [(b) R - { 3 , 5 } (c) { 3 , 5 } (d) [3 , 5 [

SOL

⟹ the domain of the function f : f (x) = $\frac{x}{\sqrt[3]{x}-2}$ is ……………

(a) R (b) R - {2} (c) R - {0 , 2} (d) R - {8}

SOL

⟹ if the domain of the function f : f (x) = $\sqrt{\frac{x}{x+1}}$ is ……

(a)] -1 , 0 [(b) R - {-1} (c)] -1 , 0] (d) R -] -1 , 0 [

SOL

⟹ the domain of the function f : f (x) = $\frac{\sqrt{x-2}}{x-3}$ is ……

(a) R (b) {3} (c) [2 , ∞ [(d) [2 , ∞ [- {3}

SOL

⟹ the domain of the function ƒ : ƒ (x) = $\dfrac{1}{\sqrt[3]{x^2 - 5\,x - 6}}$ is ....

(a) R - {5} (b) R - {6} (c) R - { 1 , - 6 } (d R - { - 1 , 6 }

SOL

....

⟹ the domain of the function ƒ : ƒ (x) = $\dfrac{1}{\sqrt[3]{x^2 + 9\,x + 8}}$ is ....

(a) R - {-8 , -1} (b) R - [-8 , -1] (c)] -8 , - 1 [(d) { - 8 , -1 }

SOL

....

⟹ the domain of the function ƒ : where

$$f(x) = \begin{cases} x - 3 & , \quad -2 < x < 6 \\ 5 & , \quad 6 < x < 12 \end{cases}$$ is ....

(a) [-2 , 12] (b)] -2 , 12] (c)] -2 , 12 [(d) [-2 , 12] – { 2 }

SOL

....

⟹ if : $f(x) = \begin{cases} 2x+3 & , \quad x < 6 \\ x^2 - 1 & , \quad x \geq 6 \end{cases}$, Then : f (6) = …………

(a) 6 (b) 15 (c) 35 (d) 11

SOL

……………………………………………………………………………………………

⟹ if : $f(x) = \begin{cases} x^2 - 4 & , \quad -1 < x < 4 \\ x+1 & , \quad 4 < x < 8 \\ zero & , \quad 8 \leq x < 12 \end{cases}$, Then : f (10) - f (5) + f (2) = …………

(a) 6 (b) 5 (c) - 6 (d) -5

SOL

……………………………………………………………………………………………

⟹ if : $f(x) = \begin{cases} -4x+3 & , \quad x < 3 \\ -x^3 & , \quad 3 \leq x \leq 8 \\ 3x^2 + 1 & , \quad x > 8 \end{cases}$, Then : f (10) = …………

(a) - 37 (b) - 1000 (c) 301 (d) 43

SOL

……………………………………………………………………………………………

⟹ the domain of the function f : f (x) = $\begin{cases} -2 & , \ x < 2 \\ 3 & , \ x > 2 \end{cases}$ is ……………

(a) R (b) R - {3} (c) R - {-2} (d) R - {2}

SOL

……

⟹ the domain of the function f : f (x) = $\begin{cases} x^2 + 2 & , \ -2 \le x < 0 \\ x - 1 & , \ x > 0 \end{cases}$ is …………

(a) R (b) [-2 , 0 [(c) [-2 , ∞ [- {0} (d) [-2 , ∞ [

SOL

……

⟹ the domain of the function f : f (x) = $\begin{cases} x & , \ 0 \le x \le 1 \\ 2 - x & , \ 1 < x \le 2 \end{cases}$ is ……………

(a) R - {1} (b) [0 , 2] (c) R - { 0 , 2 } (d)] 0 , 2 [

SOL

……

Identifying the (domain – range)

Of the function From its graph

from the graph :

- **the domain is the set of the x-coordinates**

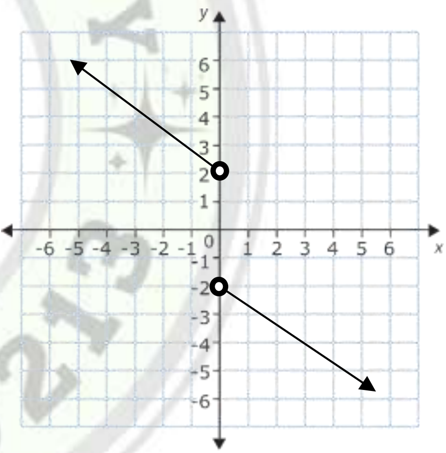
Of all the points that lie on the curve of the function

- **the range is the set of the y-coordinates**

Of all the points that lie on the curve of the function

⟹ the opposite figure represents

function of x , its range is ……



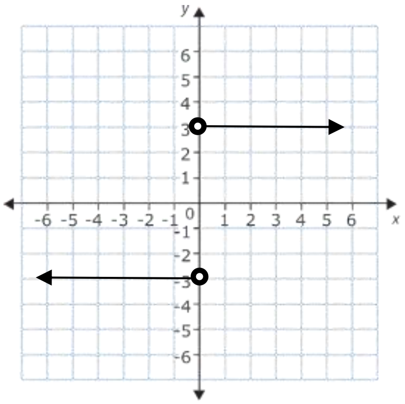
(a) R - [-2 , 2 [(b) R -] - 2 , 2 [

(c)] -2 , 2 [(d) R - { -2 , 2 }

⟹ the opposite figure represents function of x , its range is ……

(a) R - { 1 , -1 }　　(b) { -3 , 3 }

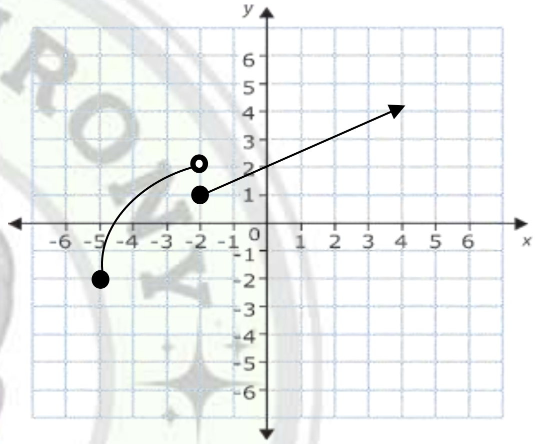
(c) { 0 }　　(d) R - {0}



⟹ the opposite figure represents function of x , its domain is ……

(a) R - {- 5 , - 2 }　　(b)] - 5 , - 2 [

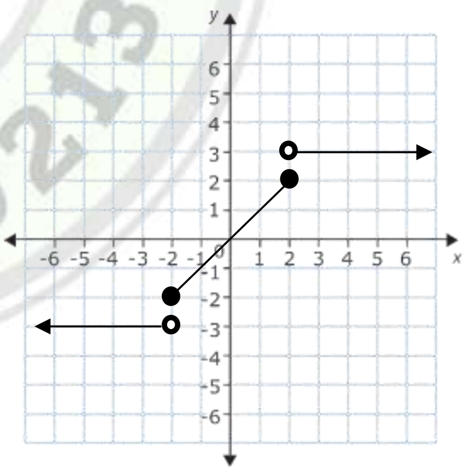
(c) [- 5 , ∞ [　　(d) [- 5 , ∞ [- {-2}



⟹ the opposite figure represents function of x , its range is ……

(a) R　　(b) [-2 , 2]

(c) [-2 , 2] ∪ {-3 , 3 }　　(d)] -3 , 3 [



Discussing the monotony

Of a function From its graph

● Discussing of the monotony of a function meaning identifying the intervals on which the function is increasing , the intervals on which the function is decreasing And the intervals on which the function is constant

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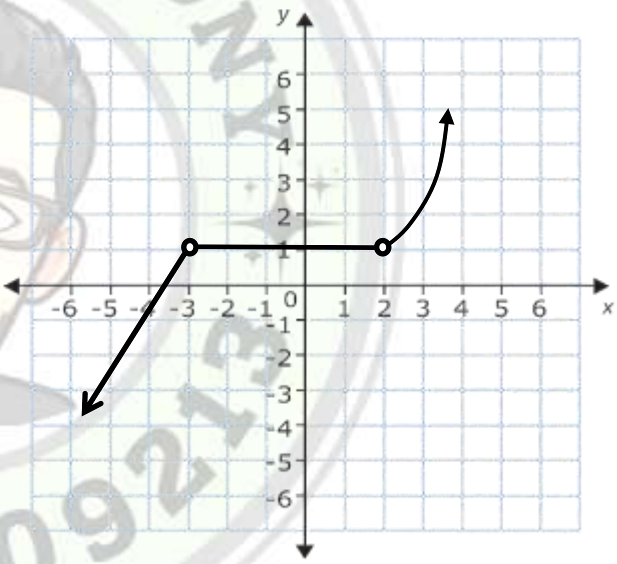


⟹ the opposite figure represents function of x

➔ its domain is …………

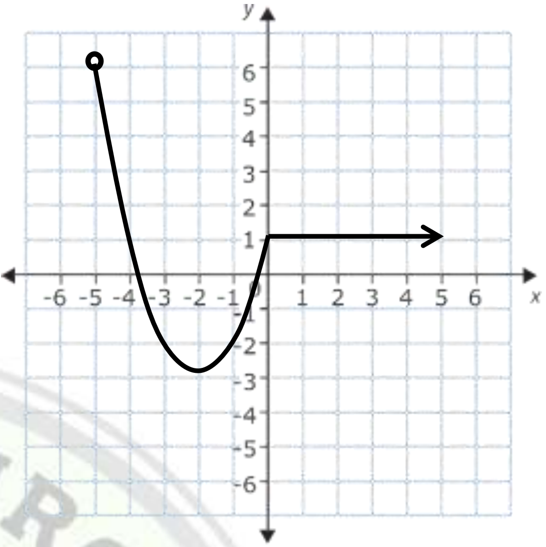
➔ its range is …………

➔ its monotony is

- constant on : …………
- increasing on : …………

⟹ the opposite figure represents

function of x



➔ its domain is …………

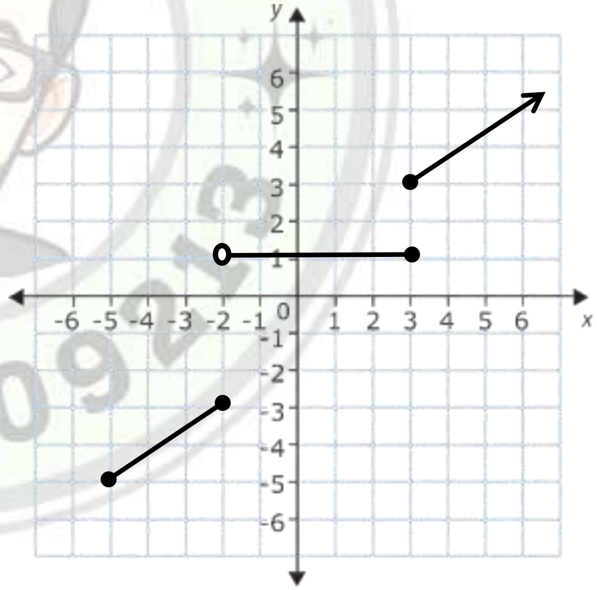
➔ its range is …………

➔ its monotony is

- decreasing on : …………
- increasing on : …………
- constant on : …………

⟹ the opposite figure represents

function of x



➔ its domain is …………

➔ its range is …………

➔ its monotony is

- increasing on : …………
- increasing on : …………
- constant on : …………

Operation on functions

● in all the operations on the functions, the domain of the resulting function equals the intersection of the domains of the two functions except the zeroes of the divisor in the division operation .

⟹ if $f : \mathbb{R}^{+} \to \mathbb{R}$ where $f(x) = 2x^2 - 4x + 2$ and $g :]-\infty, 3] \to \mathbb{R}$ where $f(x) = x - 1$. Find : [1] $(f - g)(x)$ [2] $(f \div g)(x)$

SOL

⟹ if : f (x) = $\frac{2x-1}{x+4}$, g (x) = $\frac{2x+8}{x}$. find : (f ÷ g) (x) , (f ÷ g) (2)

SOL

....

....

....

....

....

⟹ if : f (x) = $\sqrt{2x-4}$, g (x) = $\sqrt[3]{x-5}$

then the domain of : (f ÷ g) (x) is ....

SOL

....

....

....

....

⟹ if : f (x) = $\sqrt{x-3}$, g (x) = $\sqrt{x+3}$

then the domain of : (f + g) (x) is ……………….

SOL

………

………

………

………

⟹ if : f (x) = $\sqrt{x-2}$, g (x) = $\sqrt{6-x}$

then the domain of : (f + g) (x) is ………

SOL

………

………

………

………

⟹ the domain of function (f . g) where :

$f(x) = x^2 - 1$, $g(x) = \frac{x+2}{2x}$ is ………

SOL

………

………

………

………

⟹ if : $f(x) = x^2 - 1$, $g(x) = x + 1$

then : $(2f + 3g)(1)$ is ………

SOL

………

………

………

………

⦿ Multiple choice questions :

⟹ the domain of the function f : f (x) = $\sqrt{x-2} - \sqrt{7-x}$ is …………..

(a) [2 , 7] (b)] 2 , 7 [(c) R - [2 , 7] (d) R -] 2 , 7 [

SOL

………

………

⟹ if : f (x) = $\sqrt[3]{x-1}$, g (x) = $\sqrt{x-1}$, then the domain of (f – g) = ……………

(a) [-1 , ∞ [(b) [1 , ∞ [(c) R (d)] -∞ , 1]

SOL

………

………

⟹ the domain of the function f : f (x) = $\sqrt{x-1} + \sqrt{x+2}$ is …………..

(a) [-2 , ∞ [(b) [1 , ∞ [(c)] 1 , ∞ [(d)] -2 , ∞ [

SOL

………

………

⟹ the domain of the function f : f (x) = $\sqrt{x+2} - \sqrt{3-x}$ is ....

(a) R　　(b) R - [-2 , 3]　　(c) [-2 , 3]　　(d)] -2 , 3 [

SOL

....

....

⟹ the domain of the function f : f (x) = $\frac{x-1}{x^2-1} + \frac{1}{x+2}$ is ....

(a) R - { 1 , -1 , -2 }　　(b) R - { 1 , -2 }　　(c) R - { 1 , -1 , 2 }　　(d) { 1 , 2 }

SOL

....

....

⟹ the domain of the function f : f (x) = $\sqrt{x-4}\ \sqrt[3]{x-2}$ is ....

(a)] 4 , ∞ [　　(b) [2 , ∞ [　　(c) R　　(d) [4 , ∞ [

SOL

....

....

⟹ if : f , g are two real functions where f $(x) = \frac{x-2}{x^2-3x+2}$, g $(x) = x - 3$

then : $(\frac{f}{g})(3)$ = ……………….

(a) 1 (b) 2 (c) 4 (d) 8

SOL

………

………

⟹ if : f , g : R ⟶ R where f $(x) = 3x + 1$ and (f + g)$(x) = x^3 + 2x - 1$

then : g (-1) = ……………….

(a) -2 (b) -1 (c) zero (d) 2

SOL

………

………

⟹ if : ƒ (x) = $\frac{1}{x}$, g (x) = $\sqrt{x}$, then the domain of (ƒ . g) = ……………….

(a) R - { 0 }   (b) R   (c) R^+ (d) [4 , ∞ [

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ if ƒ : R^+ ⟶ R where ƒ (x) = x^2 - 7 , g : [-3 , 2] ⟶ R where g (x) = x^2 + 3

then : (ƒ – g)(-5) = ……………….

(a) -5   (b) -10   (c) -20   (d) undefined

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ if ƒ : R ⟶ R where ƒ (x) = x^2 - x , g : R ⟶ R where g (x) = 3 x - 2

then : (ƒ . g – 6g)(2) = ……………….

(a) 24   (b) 40   (c) 16   (d) -16

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ if $f : R^{+} \rightarrow R$ where $f(x) = x - 5$, $g : [-1, 5] \rightarrow R$ where $g(x) = x - 2$

then : $(\frac{f}{g})(1)$ = ……………….

(a) 1 (b) 2 (c) 4 (d) 8

SOL

⟹ the domain of the function (f . g) , $f(x) = x^2 - 4$, $g(x) = x + 1$ is ……….

(a) R - { 2 , -2 , -1 } (b) { 2 , -2 , -1 } (c) R (d) R - { 4 , 1 }

SOL

⟹ if $f(x) = \sqrt[3]{x - 3}$, $g(x) = \sqrt{3 - x}$. then the domain of $(\frac{f}{g})$ is ………….

(a)] 3 , ∞ [(b) [3 , ∞ [(c)] - ∞ , 3 [(d)] - ∞ , 3]

SOL

⟹ if $f : R^+ \rightarrow R$ where $f(x) = x - 5$, $g : [-1, 5] \rightarrow R$ where $g(x) = x - 2$

then the domain of $\left(\frac{f}{g}\right)$ is ……………….

(a) $]0, 5]$ (b) $]0, 5] - \{2\}$ (c) $[-1, \infty[$ (d) R

SOL

………

………

⟹ if $f_1 : [-2, 3] \rightarrow R$ where $f_1(x) = x$, $f_2 : [0, 4[\rightarrow R$ where $f_1(x) = x^2$

then the domain of $\left(\frac{f_1}{f_2}\right)$ is ……………….

(a) $[-2, 4[$ (b) $[-2, 4[- \{0\}$ (c) $]0, 3]$ (d) $[0, 3]$

SOL

………

………

Composition on functions

- if : (ƒ) and (g) are two functions,and range of (g)

intersection domain of (f) is not equal to ∅

then:the composition of the function (ƒ)

with the function (g) gives a new function ($ƒ \circ g$)

which is read as (ƒ composed g) or (ƒ after g)

Where : $(ƒ \circ g) (x) = ƒ \left(g (x) \right)$

and its domain = $\{ x : x \in$ domain of g , $g (x) \in$ domain of $f \}$

🔔 **Remark …**

- $(ƒ \circ g)(x) \neq (g \circ ƒ)(x)$

i.e. operation of composition of functions is not commutative

---

⟹ if : ƒ (x) = 2 x + 1 , g (x) = $x^2 - 3$

Then : (ƒ ○ g)(2) + (g ○ ƒ)(1) = ………

SOL

……

……

……

……

⟹ if : ƒ is a linear function and (ƒ ○ ƒ)(x) = 4 x + 15

Find : ƒ (x)

SOL

……

……

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……

⟹ if : $f(x) = 2x + 1$, $(f \circ g)(x) = 2x^2 - 6x - 1$

Find : $g(x)$

SOL

⟹ if : $f(x) = \sqrt{2x - 6}$, $g(x) = \sqrt{1 - x}$

Find the domain of : $(f \circ g)$

SOL

⟹ if : ƒ (x) = $\sqrt{x-1}$, g (x) = $x^2 - 3$

Find : (ƒ ○ g) (x) in the simplest form , showing the domain

Then find : (ƒ ○ g) (2)

SOL

◉ Multiple choice questions :

⟹ if : ƒ (x) = $\sqrt{x-7}$, g (x) = x^2 . then : (ƒ ○ g) (4) = ...

(a) 9 (b) 7 (c) 3 (d) 11

SOL

....

....

⟹ if : ƒ (x) = 5 x + 4 , g (x) = 2 – x . then : (ƒ ○ g) (-2) + (ƒ ○ g) (5) = ...

(a) 24 (b) -27 (c) 49 (d) -3

SOL

....

....

⟹ if : ƒ (x) = $\sqrt{x}$, g (x) = x^2 . then : the domain of (g ○ ƒ) = ...

(a)] - ∞ , 0] (b) [0 , ∞ [(c) R^+ (d) R

SOL

....

....

⟹ if : $f(x) = \frac{3x+2}{2x-3}$, then : $(f \circ f)(x)$ = ………

(a) x (b) - x (c) f (x) (d) - f (x)

SOL

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⟹ if : $f(x) = \sqrt[3]{x}$, $g(x) = x^3$, then : $(f \circ g)(x)$ = ……………

(a) x (b) x^3 (c) $\sqrt[3]{x}$ (d) $|x|$

SOL

……………………………………………………………………………………………………

……………………………………………………………………………………………………

⟹ if : $f(x) = \sqrt{x-2}$, $g(x) = \sqrt{6-x}$, then the domain of : $(f \circ g)$ is ………

(a) $]-\infty, 2]$ (b) $[2, \infty[$ (c) $]-\infty, 2[$ (d) R

SOL

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⟹ if : ƒ (x) = $x^2 + 6$, g (x) = 3 x , and : (ƒ ○ g) (x) = 42 , then : x = ………

(a) ± 4 (b) - 4 (c) 2 (d) ± 2

SOL

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⟹ if : ƒ (x) = x + 2 , g (x) = $\frac{x-k}{3x-1}$, and : (ƒ ○ g) (1) = (g ∘ ƒ) (0) , then : k = ………

(a) 7 (b) 5 (c) 1 (d) $\frac{7}{2}$

SOL

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⟹ if : ƒ (x) = $\sqrt{x+5}$, g (x) = x^2 . then : (ƒ ○ g) (2) = ……………

(a) 7 (b) 3 (c) 4 (d) 9

SOL

……………………………………………………………………………………………

……………………………………………………………………………………………

⟹ if : $f(x) = x^2 + 6$, $g(x) = 3x$, then $(f \circ g)(3) =$ ………

(a) 75 (b) 87 (c) 90 (d) 100

SOL

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⟹ if : $g(x) = \frac{x}{x-1}$, then : $(g \circ g)(1) =$ ………

(a) 1 (b) zero (c) -1 (d) undefined

SOL

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…………………………………………………………………………………………………

⟹ if : $f(x) = x - 3$, $g(x) = x^2$, then $(f \circ g)(x) =$ ………

(a) $(x-3)^2$ (b) $x^2 - 3$ (c) $x^2 + 3$ (d) $\sqrt{x-3}$

SOL

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⟹ if : $f(x) = 2x + 5$, $g(x) = x^2$, then : $f\left(g(x)\right) =$ ………

(a) $(2x + 5)^2$ (b) $2x^2 + 5$ (c) $(2x^2 + 5)^2$ (d) $2x^2 + 10$

SOL

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⟹ if : $f(x) = \sqrt{x}$, $g(x) = x^2$. then : the domain of $(f \circ g) =$ ……………

(a) $]-\infty, 0]$ (b) $[0, \infty[$ (c) R^+ (d) R

SOL

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⟹ if : $f(x) = 3x + 2$, $g(x) = 2x + k$ and $(f \circ g)(x) = (g \circ f)(x)$, then : $k =$ ………

(a) 1 (b) 2 (c) 3 (d) 4

SOL

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⟹ if : $f(x) = 2x$, $g(x) = \frac{1}{2x}$, $f\left(g(x)\right) = 6$, then : x = ...

(a) $\frac{1}{6}$ (b) 6 (c) 4 (d) $\frac{1}{12}$

SOL

....

....

⟹ if : $f(x) = \frac{1}{x}$, $g(x) = x^2 - 1$, then : the domain of $(f \circ g)$ = ...

(a) { 0 , 1 , -1 } (b) R - { 0 , 1 , -1 } (c) R - { 1 , -1 } (d) [-1 , 1]

SOL

....

....

⟹ if : $f(x) = 4x - 5$, $g(x) = 3^x$, then : $(f \circ g)(2)$ = ...

(a) 3 (b) 9 (c) 27 (d) 31

SOL

....

....

⟹ if : ƒ (x) = x – 1 , g (x) = $\sqrt{x}$. then : the domain of (g ○ ƒ) = ……………

(a) R (b)] - ∞ , 1 [(c) [1 , ∞ [(d) R - {1}

SOL

………

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⟹ if : ƒ (x) = x + 1 , g (x) = $x^2 - 1$, then the solution set of

the equation : g $\left(ƒ(x)\right)$ = ………..

(a) { 0 , 2 } (b) { 1 , -1 } (c) { 0 , -2 } (d) { 2 , -2 }

SOL

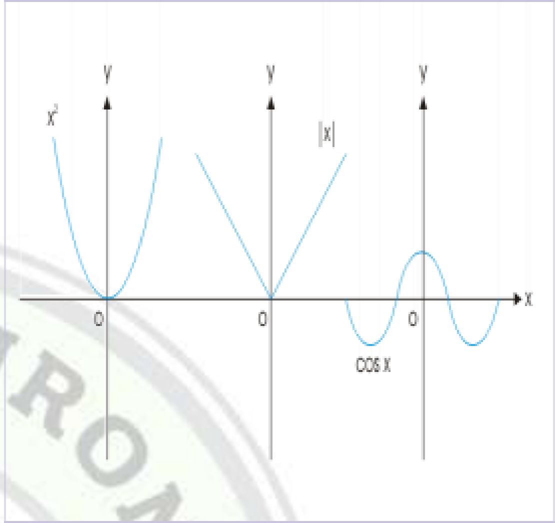
………

………

Some Properties Of Functions

Even function and Odd function :

Even function :



the function ƒ is said to be even

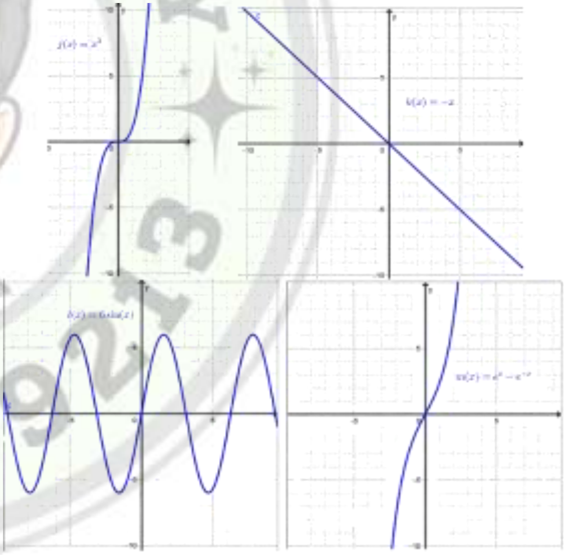
if : ƒ (-x) = ƒ (x)

for each x , - x ∈ the domain .

So : the curve of the even

function is symmetric about y-axis

Odd function :



the function ƒ is said to be odd

If : ƒ (-x) = - ƒ (x)

for each x , - x ∈ the domain .

So : the curve of the even

function is symmetric about the origin point

■ Determine which of the function is even , odd or otherwise

F $f(x) = x^{2} - 2$

SOL

....

....

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....

F $f(x) = 2x^{3} + 1$

SOL

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....

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F $f(x) = 2x^3 - 3x$

SOL

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F $f(x) = \sqrt{x - 1}$

SOL

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F ƒ (x) = $\frac{x - \sin 3x}{1 + x^2}$

SOL

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F ƒ (x) = $\begin{cases} -\frac{1}{x} & , \quad x < 0 \\ \frac{1}{x} & , \quad x > 0 \end{cases}$

SOL

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⦿ Multiple choice questions :

F the folowing functions are even except ……………..

(a) ƒ (x) = sin x (b) ƒ (x) = cos x (c) ƒ (x) = 3 (d) ƒ (x) = $x^2 - 1$

SOL

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F if : ƒ is an odd function , ƒ (3) = 5 . Then which of the following points lies on the curve of ƒ ?

(a) (3 , -5) **(b) (-3 , -5)** **(c) (-3 , 5)** **(d) (5 , 3)**

SOL

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……………………………………………………………………………………………………

F if : ƒ is an even function , ƒ (1) = 3 . Then which of the following points lies on the curve of ƒ ……………..

(a) (1 , -3) **(b) (-1 , -3)** **(c) (-1 , 3)** **(d) (3 , 1)**

SOL

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F if : ƒ is an odd function , a ∈ the domain of ƒ

Then : ƒ (a) – ƒ (-a) = ……….

(a) zero (b) ƒ (a) (c) 2 ƒ (a) (d) ƒ (2a)

SOL

………

………

F if : ƒ is an even function , a ∈ the domain of ƒ

Then : ƒ (a) – ƒ (-a) = ……….

(a) zero (b) ƒ (a) (c) 2 ƒ (a) (d) ƒ (2a)

SOL

………

………

F if : ƒ is an even function over [a , b] , then : b = …………

(a) a (b) - a (c) 2 a (d) a^2

SOL

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F if : ƒ is an odd function over [a , b] , $a < 0$

then : b = …………

(a) a (b) - a (c) 2 a (d) a^2

SOL

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F the function ƒ : ƒ (x) = $x^4 - 3x^2 + 1$

is symmetric about ………………

(a) the x-axis (b) the y-axis (c) the origin point

SOL

………

F the function ƒ : ƒ (x) = $x^3 + 2x$

is symmetric about ……………

(a) the x-axis (b) the y-axis (c) the origin point

SOL

………

F the function ƒ : ƒ (x) = $2x^3 + \sin x - x$ is …………………

(a) odd (b) even (c) neither odd nor even

SOL

………

………

………

F the function $f : f(x) = x \tan x - 1$ is …………………

(a) odd (b) even (c) neither odd nor even

SOL

………

F the function $f : f(x) = \sin 2x$

is symmetric about the point …………………

(a) $(0, 0)$ (b) $(2, 0)$ (c) $(-2, 0)$ (d) $(-2, 2)$

SOL

………

F if : $f(x) = x^2 - a x - 4$ is an even function

then : a = …………………

(a) 2 (b) 1 (c) zero (d) 4

SOL

………

F if : $f(x) = x^3 - 3x - a$ is an odd function

then : a = …………………

(a) 3 (b) 1 (c) zero (d) - 3

SOL

………

F if : $f(x) = a x^3 + b$ is an odd function , and the curve of the function passes through

the point (2 , 8) . then : $a + b^2$ = …………………

(a) - 1 (b) 1 (c) zero (d) 5

SOL

………

………

● One – to – one function :

Definition :

The function ƒ : x → y is called one – to – one function

if each : a , b ∈ x , ƒ (a) = ƒ (b) , then : a = b

Or for each : a , b ∈ x , a ≠ b , ƒ (a) ≠ ƒ (b)

And this means there are not two elements in the domain

Of the one–to–one function having the same image .

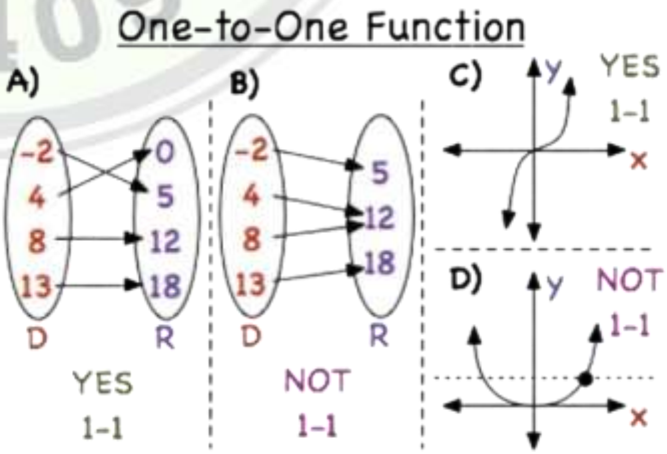
The horizontal line test :

if : there exist a horizontal line

Intersects the curve of the function

At more than one point

Then the function is not one – to – one


One-to-One Function
A)
-2
4
8
13
0
5
12
18
D
R
YES
1-1
B)
-2
4
8
13
5
12
18
D
R
NOT
1-1
C)
y
x
YES
1-1
D)
y
x
NOT
1-1

Then
- the even function are not one – to – one
- the odd function could to be one – to – one or not

- **Determine which of the function is one – to - one :**

F $f(x) = x^3 - 4$

SOL

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F $f(x) = x^2 + 1$

SOL

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F ƒ(x) = $x^2 - 2x$

SOL

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F ƒ (x) = $\frac{2x-5}{5x+2}$

SOL

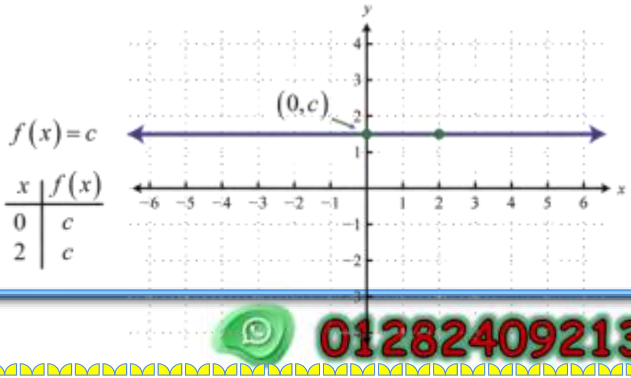
Graphical representation of functions

● Representing the costant function :

■ ƒ : R → R , ƒ (x) = c where c ∈ R

☛ the range = {c}

☛ the function is even



☛ the function is not one – to – one

F if : f (x) = 3 , then the domain of the function f is ……………

(a) R (b) R^+ (c) {3} (d) R - {3}

F if : f (x) = 2 , then the range of the function f is ………

(a) R (b) R^+ (c) {2} (d) R - {2}

F the function f where f (x) = $\begin{cases} -3 & , x < 0 \\ 3 & , x > 0 \end{cases}$

is symmetric about the point ………

(a) (3 , 0) (b) (- 3 , 0) (c) (0 , 0) (d) (-3 , 3)

● Representing the linear function :

■ its form : f (x) = a x + b

is represented graphically by a stright line :

☛ cutting the x – axis at The point ($\frac{-b}{a}$, 0)

☛ cutting the y – axis at The point (0 , b)

☛ its slope = a

F Represent graphically the function f : f (x) = $\frac{x^2 + x}{x}$

SOL

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F Represent graphically the function f : f (x) = 3 – 2 x

Where : x ∈ [- 1 , 4 [

SOL

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Notes …

☛ in the linear function

- if : a $>$ 0 then : the function is increasing
- if : a $<$ 0 then : the function is decreasing

⦿ Multiple choice questions :

F the range of the function ƒ : ƒ (x) = $\frac{2x^2 - 2}{x^2 - 1}$ is ………

(a) R (b) R – { 2 } (c) R – { 1 , -1 } (d) { 2 }

SOL

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……………………………………………………………………………………

F the range of the function ƒ : ƒ (x) = $\frac{2x^2 - 2}{x - 1}$ is ………

(a) R (b) R – { 2 } (c) R – { 1 } (d) R – { 4 }

SOL

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F the domain of the function ƒ : ƒ (x) = $\frac{x^2 - 1}{x + 1}$ is ………

(a) R (b) R – {-1} (c) R – { 1 } (d) R – { 1 , -1 }

SOL

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● Representing the quadratic function :

■ its form : ƒ (x) = (x + a)^2 + b

is represented graphically by a curve :

☛ its vertex (– a , b)

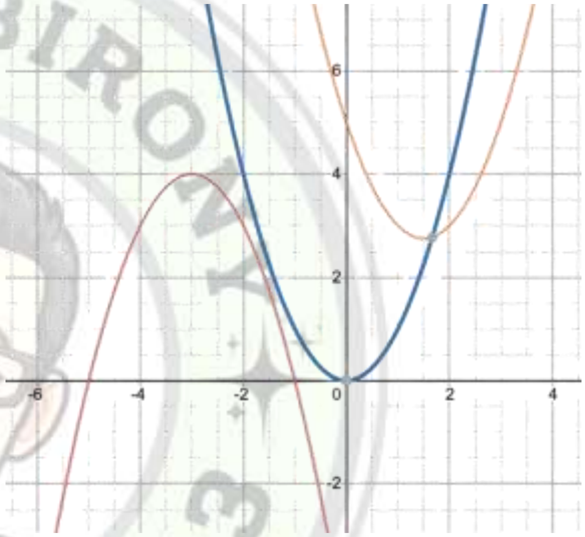
▶ range = [b , ∞ [or] - ∞ , b]

▶ decreasing 0n] - ∞ , -a [

▶ increasing 0n] -a , ∞ [

▶ the function is even if its vertex (0 , y)

▶ the function is not one-to-one



☛ the simplest form : ƒ (x) = x ^{2} , where : a = b = 0

☛ its vertex (0 , 0)

F Represent the function ƒ : ƒ (x) = (2 – x)^2 + 1

SOL

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☛ the curve of the function ƒ (x) = (x – 2)^2 + 1

is the same curve ƒ (x) = x ^{2}

by ahorizontal translation (2) unit in the direction $\overrightarrow{o\,x}$

Then a vertical translation (1) unit in the direction $\overrightarrow{o\,y}$

Note : (a - x)^2 + b = (x - a)^2 + b

◉ Multiple choice questions :

F the point of the vertex of the curve of the function ƒ : $ƒ(x) = (2 - x)^2 - 4$ is …………

(a) (2 , 4) (b) (-2 , 4) (c) (-2 , -4) (d) (2 ,-4)

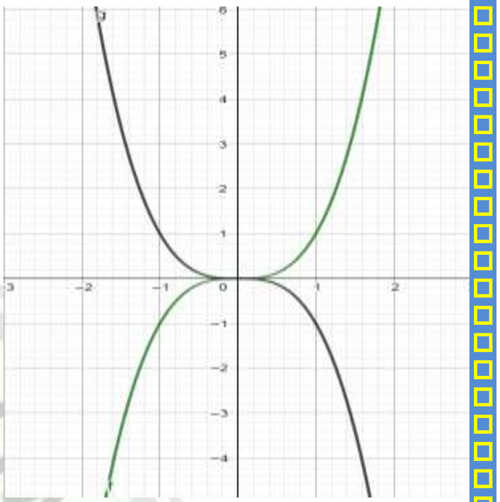
F the range of the function ƒ : $ƒ(x) = -(3 - x)^2$ is ……………

(a) $]-\infty , 3]$ (b) $[0 , \infty[$ (c) $]-\infty , 0[$ (d) $[3 , \infty[$

F the axis of symmetry for the function ƒ : $ƒ(x) = x^2 - 4$ is …………

(a) $x = 0$ (b) $x = 4$ (c) $y = 4$ (d) $y = -4$

⊙ Representing the cube function :



■ its form : $f(x) = (x + a)^3 + b$

is represented graphically by a curve :

☛ its vertex (– a , b)

▶ range = R

▶ increasing or decreasing 0n R

▶ the function is odd if its vertex (0 , 0)

▶ the function is one-to-one

☛ the simplest form : $f(x) = x^3$, where : a = b = 0

☛ its vertex (0 , 0)

F Represent the function f : f (x) = $(x - 1)^3 - 2$

SOL

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☛ the curve of the function f (x) = $(x - 1)^3 - 2$

is the same curve f (x) = x^3

By ahorizontal translation (1) unit in the direction $\overrightarrow{o\,x}$

Then a vertical translation (2) unit in the direction $\overrightarrow{o\,y`}$

🔔 **Note :** $(a - x)^3 + b = -(x - a)^3 + b$

◉ **Multiple choice questions :**

F the symmetric point of the function f : f $(x) = x^3 - 1$ is ....

(a) (0 , 0) (b) (0 , 1) (c) (0 , -1) (d) (1 , - 1)

F the domain of the function f : f $(x) = (x - 3)^3$ is ....

(a)] -∞ , 3] (b) R (c) [0 , ∞ [(d) [3 , ∞ [

F the symmetric point of the function f : f $(x) = -(x - 3)^3 - 1$ is ....

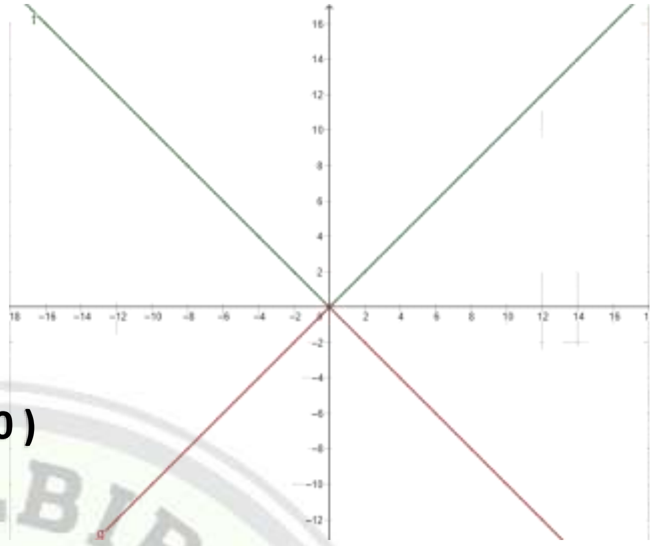
(a) (3 , 1) (b) (3 , -1) (c) (-3 , 1) (d) (-3 , 1)

F if the function f : f $(x) = -(2 - x)^3$, then : f (0) = ...

(a) zero (b) 2 (c) - 2 (d) - 8

Representing the absolute function :

■ its form : $f(x) = |x + a| + b$



☛ its vertex (– a , b)

▶ range = [b , ∞ [

▶ decreasing on] - ∞ , - a [

▶ increasing on] - a , ∞ [

▶ the function is even if its vertex is (0 , 0)

▶ the function is not one-to-one

☛ the simplest form : $f(x) = |x|$, where : a = b = 0

☛ its vertex (0 , 0)

F Represent the function f : $f(x) = |2 - x| + 3$

SOL

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☛ the curve of the function f(x) = | $x - 2$ |+ 3

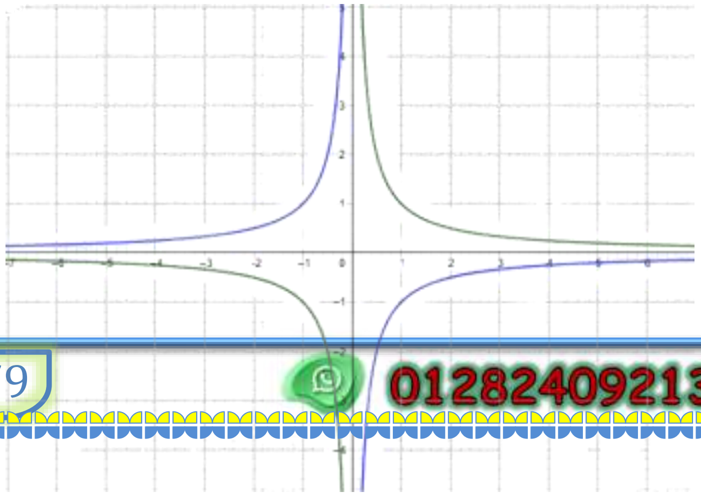
is the same curve f (x) = | x |

by ahorizontal translation (2) unit in the direction $\overrightarrow{o\,x}$

Then a vertical translation (3) unit in the direction $\overrightarrow{o\,y}$

🔔 **Note :** | a $-$ x | + b = | x $-$ a | + b

● Representing the rational function :



■ its form : f (x) = $\frac{1}{x+a}$ + b

☛ its symmetric point (– a , b)

▶ domain = R - {-a}

▶ range = R - {b}

▶ decreasing on R - {- a}

▶ the function is odd if its symmetric point is (0 , 0)

▶ the function is one-to-one

☛ the simplest form : f $(x) = \frac{1}{x}$, where : a = b = 0

☛ its symmetric point (0 , 0)

F Represent the function f : f (x) = $\frac{1}{x-2} - 3$

SOL

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☛ the curve of the function ƒ (x) = $\frac{1}{x-2} - 3$

is the same curve ƒ (x) = $\frac{1}{x}$

by ahorizontal translation (2) unit in the direction $\overrightarrow{o\,x}$

Then a vertical translation (3) unit in the direction $\overrightarrow{o\,y`}$

🔔 **Note :** $-\frac{1}{a-x}$ + b = $\frac{1}{x-a}$ + b

F Represent the function ƒ : ƒ (x) = $\frac{1}{|x-2|} + 1$

SOL

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F Represent the function f : f (x) = $\left|x^2 - 1\right|$

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F Represent the function f : f (x) = $\left|x^3\right|$ + 2

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Note : $|x^3| + 2 = x^2 + 2 = x^2\,|x| + 2 = x\,|x^2| + 2$

Solving absolute value eguations

(graphically – algebraically)

F find the solution set of the equation : $|x - 1| = 3$ in R

graphically and algebraically

SOL

first : graphically

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second : algebraically

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F find the solution set of the equation : $|2x+1|$ = x + 2

in R graphically and algebraically

SOL

first : graphically

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second : algebraically

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F find the solution set of the equation :

$|2x-1|$ = $|x+1|$ in R graphically and algebraically

SOL

first : graphically

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second : algebraically

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Solving absolute value inequalities

(graphically – algebraically)

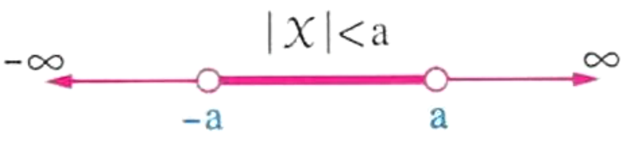
⦿ graphically …

For any two functions f and g

- $f(x) < g(x)$ ▶ S.S. = $]a, b[$
- $f(x) \leq g(x)$ ▶ S.S. = $[a, b]$
- $f(x) > g(x)$ ▶ S.S. = $\mathbb{R} - [a, b]$
- $f(x) \geq g(x)$ ▶ S.S. = $\mathbb{R} -]a, b[$

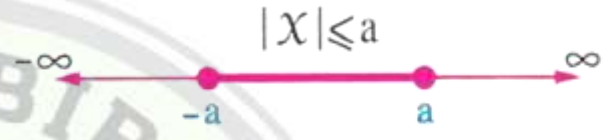
◉ algebraically …

■ $|x| < a$. then : $-a < x < a$



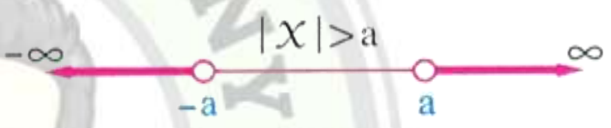
➔ S.S. =] - a , a [

■ $|x| \leq a$. then : $-a \leq x \leq a$



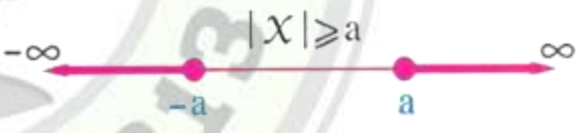
➔ S.S. = [- a , a]

■ $|x| > a$. then : $x > a$ or $x < -a$



➔ S.S. = R - [- a , a]

■ $|x| \geq a$. then : $x \geq a$ or $x \leq -a$



➔ S.S. = R -] - a , a [

F find in R the S.S. of : $|2x + 1| < 5$

SOL

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F find in R the S.S. of : $\frac{1}{|\,2\,x-1\,|} \leq 4$

SOL

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F find in R the S.S. of : $\sqrt{4\,x^2 - 4\,x + 1} \leq 7$

SOL

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■ Determine the domain :

F f (x) = $\frac{x-1}{|x|-2}$

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F f (x) = $\frac{x}{|x-3|-5}$

SOL

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F f (x) = $\frac{x+2}{4+|x-1|}$

SOL

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F f (x) = $\sqrt{|x| - 5}$

SOL

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F f (x) = $\frac{3x}{\sqrt{|x-3|} - 4}$

SOL

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Rational exponents

⦿ The n^{th} root :

The equation : $x^n = a$, $a \in R$, $n \in Z^+$ has n root

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■ the properties of the n^{th} root :

- ▶ $\sqrt[n]{a^n} = |a|$ if n is an even number
- ▶ $\sqrt[n]{a^n} = a$ if n is an odd number
- ▶ $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$
- ▶ $a^{\frac{1}{n}} = \sqrt[n]{a}$
- ▶ $a^{-n} = \frac{1}{a^n}$ ▶ $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$
- ▶ $a^m \times a^n = a^{m+n}$
- ▶ $a^m \div a^n = a^{m-n}$
- ▶ $(a^m)^n = a^{m \times n}$

■ Simplify to simplest form :

F $\frac{(27)^{-3} \times (12)^{2}}{16 \times (81)^{-2}}$

SOL

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F $\frac{(9)^{4n+1} \times (4)^{2-2n}}{(3)^{9n+1} \times (48)^{1-n}}$

SOL

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F Prove that :

$$\frac{(343)^{2n-\frac{1}{3}} \times (4)^{3n+1}}{(196)^{3n} \times 4} = \frac{1}{7}$$

SOL

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Exponential equations

◉ Laws of exponents :

■ For every m , n ∈ Z and a , b ∈ R - { -1 , 0 , 1 }

⍰ if : $a^n = 1$ then : $n = 0$

⍰ if : $a^m = a^n$ then : $m = n$

⍰ if : $a^n = b^n$

then : ► $a = b$ if : n is an odd number

► $a = \pm b$ if : n is an even number

► $n = 0$ if : $a \neq b$

F if : $2^{x-1} = 16$, find : the value of x

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F if : $(5)^{x^2-1} = 125$, find : the value of x

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F if : $\left(\frac{2}{3}\right)^{x^2} = \left(3\frac{3}{8}\right)^{x+3}$, find : the value of x

SOL

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F if : $x^{2x-1} = 3^{2x-1}$, find : the value of x

SOL

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F find in R the S.S. of : $2^{x-2} + 2^{x+2} = 17$

SOL

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F find in R the S.S. of : $5^{x} + \frac{125}{5^{x}} = 30$

SOL

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F if : $f(x) = (7)^{x+1}$, then find the value of x

That satisfies : $f(2x - 1) + f(x - 2) = 50$

SOL

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F if : $f(x) = 3^{x}$, then prove that : $\frac{f(2x+2) + f(2x-1)}{5f(2x) - 7f(2x-1)} = \frac{7}{2}$

SOL

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F if : ƒ (x) = 2 ^{x}, then prove that : $\frac{f\,(x+1)}{f\,(x-1)} + \frac{f\,(x-1)}{f\,(x+1)} = \frac{17}{4}$

SOL

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◉ Multiple choice questions :

F if : $5^{x-1} = 3^{x-1}$, then : x = ………

(a) 5 (b) -1 (c) 1 (d) zero

SOL

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F if : $7^{x+1} = 3^{2x+2}$, then : 5^{x+1} = ………

(a) 5 (b) -1 (c) 1 (d) zero

SOL

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F if : $5^{x} = 2$, then : $25^{x} =$ ………

(a) 10 (b) 625 (c) 4 (d) 2

SOL

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F if : $x^{\frac{2}{3}} = 4$, then : x = ………

(a) 8 (b) ± 8 (c) ± 4 (d) 4

SOL

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F if : $x^{\frac{3}{2}} = 64$, then : x = ………

(a) 512 (b) 16 (c) 4 (d) 2

SOL

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F the S.S. of the equation : $3^{x+1} + 3^{x} = 12$ in R is ………

(a) {0}   (b) {3}   (c) {1}   (d) {0 , 1}

SOL

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F the S.S. of the equation : $\sqrt[3]{x^2} - \sqrt[3]{x} + 2 = 0$ in R is ………

(a) {0 , 2}   (b) {0 , 3}   (c) {1 , 2}   (d) {0 , 1}

SOL

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F if $x \in R^{+}$, n is an even integer,which of the following is true ?

(a) $x^n > 0$ (b) $x^n < 0$ (c) $x^n \leq 0$ (d) $x^n = 0$

F if $x \in R^-$, n is an odd integer , which of the following is true ?

(a) $x^n > 0$ (b) $x^n < 0$ (c) $x^n \leq 0$ (d) $x^n = 0$

F $a < 0 < b < c$, then which of the following does not belong to R

(a) $\sqrt[3]{a\,b}$ (b) $\sqrt[3]{c\,b}$ (c) $\sqrt[3]{a\,b + c}$ (d) $\sqrt[6]{a\,c}$

Exponential function

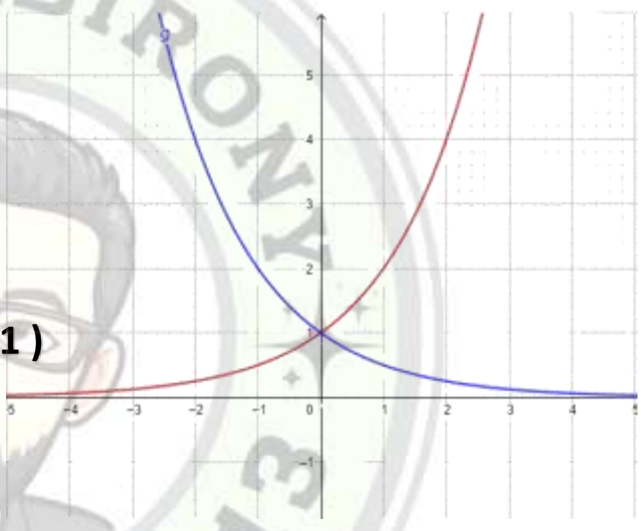
Definition :

if : $a \in R^+ - \{1\}$, then the function $f : R \rightarrow R^+$

Where : $f(x) = a^x$ is called an Exponential function

⦿ the graphical representation of the expoential function :

● the diagram of the graph of the function $f : f(x) = a^x$



- the domain = R   ■ the range = R^+
- f is increasing when : $a > 1$
- f is decreasing when : $0 < a < 1$
- f is one-to-one
- the curve passing through the point (0 , 1)
- the curve of ($y = a^{-x}$) is the image of the curve of ($y = a^x$) by reflection in y-axis
- $a^x \rightarrow \infty$ when : $x \rightarrow \infty$ where : $a > 1$
- $a^x \rightarrow 0$ when $x \rightarrow \infty$ where : $0 < a < 1$

F Graph the function $f : R \rightarrow R^+$, $f(x) = 3^x$

Taking x $\in [-3, 1]$. and from the graph

Find : (1) $f(0.5)$ (2) $f(x) = 0.19$ (3) $\sqrt[3]{3}$

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Solving the exponetial eguations graphically

F - $(\frac{1}{3})^{x-2} = -3$

SOL

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F $(2)^{x-1} = 2x - 2$

SOL

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⦿ Multiple choice questions :

F if : $f(x) = 2^{x-1}$, then : $f(x+1) =$ ………

(a) 2^{x+2} (b) 2^{x} (c) 2^{2x} (d) 2^{x+1}

SOL

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F if : $f(x) = 3^{-x}$, then : $\frac{f(x-1)}{f(x+1)} =$ ………

(a) 3 (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{9}$

SOL

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F if : $f(x) = 2^x$, then the S.S. of the equation :

$f(x+1) - f(x-1) = 24$ is …………………

(a) {0}　(b) {0 , 1}　(c) {1}　(d) {4}

SOL

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F the straight line : $y = 4$ cuts the curve of the function f where $f(x) = 2^x$ at the point ……

(a) (0 , 2)　(b) (2 , 4)　(c) (0 , 1)　(d) (0 , 4)

SOL

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F the curve of the function f : $f(x) = 3^x$

intersects the y-axis at the point …………

(a) (0 , 1)　(b) (1 , 0)　(c) (3 , 1)　(d) (1 , 3)

SOL

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F the curve of the function ƒ : ƒ (x) = 2^{x+1}

intersects the y-axis at the point ………

(a) (0 , 1) (b) (1 , 0) (c) (0 , 2) (d) (2 , 0)

SOL

……

F if : ƒ (x) = 2^{x+1} and the point (a , $\frac{1}{2}$) ∈ to the curve

Of the function ƒ , then a = …………………

(a) $\frac{1}{2}$ (b) 2 (c) - 1 (d) - 2

SOL

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F the range of the function ƒ : ƒ (x) = $(\frac{1}{2})^x$ is ………

(a) [0 , ∞ [(b)]-∞ , ∞ [(c)] 0 , ∞[(d)] 1 , ∞ [

SOL

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The inverse function

- if : ƒ is one-to-one function its domain x and its range y

Then for each element y in the range a corrosponding unique element x in the domain .

therefore it is possible to determain an inverse function from y to x which denoted by f^{-1}

Properties of the inverse function :

- $(f \circ f^{-1}) (x) = (f^{-1} \circ f) (x) = x$
- the domain of ƒ = the range of f^{-1}
- the range of ƒ = the domain of f^{-1}
- $f^{-1} (x) \neq \frac{1}{f(x)}$

- ƒ , $ƒ^{-1}$ each of them is the image of the other by reflection in the straight line : y = x

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F find the inverse function of the function ƒ : ƒ (x) = 2 x – 1

And represent each of them in the same figure

SOL

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F if : ƒ (x) = $\frac{1}{x-1}$ + 2 , find

1) the domain and the range of the function ƒ

2) ƒ $^{-1}$ (x) and determain the domain and the range of ƒ $^{-1}$

SOL

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F if : ƒ (x) = $\sqrt{x-1}$, find

1) the domain and the range of the function ƒ

2) $ƒ^{-1}$ (x) and determain the domain and the range of $ƒ^{-1}$

SOL

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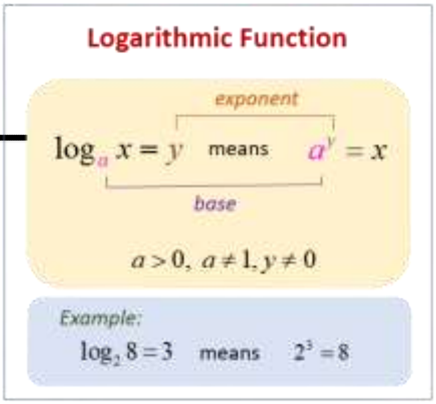
Logarithmic function

- if : a ∈ R^+ - {1} , then the function ƒ : $R^+ \to R$

Where : ƒ (x) = $\log_a x$ is called the logarithmic function



Logarithmic Function

$\log_a x = y$ means $a^y = x$

$a > 0,\ a \neq 1,\ y \neq 0$

Example:

$\log_2 8 = 3$ means $2^3 = 8$

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- y = $\log_a x$ ⟹ x = a^y

where : a ∈ R^+ - {1} , x ∈ R^+ , y ∈ R

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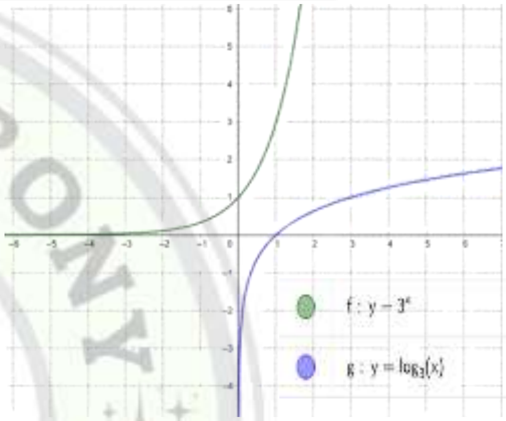
- the logarithmic function

is the inverse function

For the exponential function



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- The graphical representation of the logarithmic function ƒ

where : ƒ (x) = $\log_a x$

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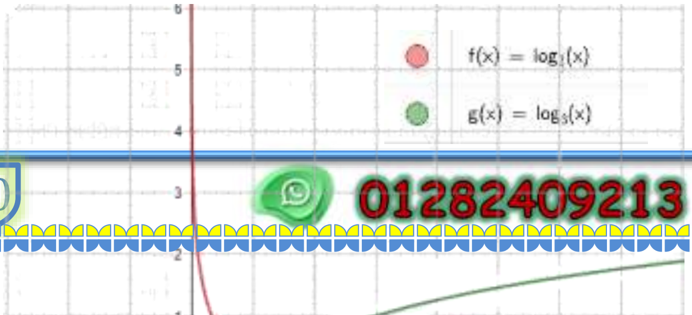
some properties of logarithmic function :

► the domain = R^+



- ► the range = R

- ► the function is increasing When : a $>$ 1

- ► the function is decreasing When : $0 < a < 1$

- ► the logarithmic function is one-to-one

- ► if : $\log_a x = \log_a y$, then : x = y

- ► all curves of the logarithmic function for any positive base $\neq 1$

pass through the point (1 , 0)

F if the curve of the function ƒ : ƒ (x) = $\log_a x$ passes through the point ($\frac{1}{16}$, 4) , find the value of a , then draw the graph of the function taking x $\in [\frac{1}{4} , 4]$, and find an approximated value to the number $\log_{\frac{1}{2}} 3.5$

SOL

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Logarithmic equation

F if : $\log_9 81\sqrt{3} = x$, find the value of x ?

SOL

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F if : $\log_{\frac{1}{2}} x = -2$, find the value of x ?

SOL

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F find in R the S.S. of : $\log_2 (x^2 - 2x) = 3$

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F find in R the S.S. of : $(\log_2 x)^2 - 5 \log_2 x = 6$

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F find in R the S.S. of : $\log_2 \log_3 (x^2 - 3x - 6) = 2$

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F find in R the S.S. of : $\log_2 \log_3 \log_2 x^2 = 0$

SOL

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Some properties of logarithms

◉ if : a ∈ R^+ - {1} , then :

- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a x\,y = \log_a x + \log_a y$
- $\log_a \frac{x}{y} = \log_a x - \log_a y$
- $\log_a x^n = n \log_a x = n \log_a |x|$ **(if n an even)**
- $\log_y x = \frac{\log x}{\log y}$
- $\log_y x = \frac{1}{\log_x y}$

F without using the calculator , find the value of :

$$\log_5 \frac{3}{5} + 2\log_5 \frac{15}{2} - \log_5 \frac{5}{36} + \log_5 \frac{5}{243}$$

SOL

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F without using the calculator , find the value of :

$$\log_2 5 \times 2\log_5 2 \times \log_3 2 \div \log_3 4$$

SOL

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F without using the calculator , prove that :

$$\frac{\log 30 - \log 6 + \log 5}{\log 12 - \log 3 + \log 25} = 1 - \log 2$$

SOL

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F using the calculator , find the value of x : $2^{x-2} = 7$

to the nearest 2 decimal digits

SOL

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F using the calculator , find the value of x : $3^{x-2} = 2 \times 5^{x+1}$

to the nearest 2 decimal digits

SOL

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F if : $x^2 + y^2 = 8xy$, then prove that : $2 \log(x + y) = 1 + \log x + \log y$

SOL

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⦿ Multiple choice questions :

F if : $\log_{\frac{1}{2}} x = -2$, then : x = ………

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) -4 (d) 4

SOL

………

F if : $\log_3 \log_2 8$ = ………

(a) 1 (b) -2 (c) 4 (d) -1

SOL

....

F if : $\log_2 x = \log_3 9$, then : x = ...

(a) 1 (b) 2 (c) 4 (d) 3

SOL

....

....

F if : $\log_2 \sqrt{x-3} = \frac{1}{2}$, then : x = ...

(a) 7 (b) 5 (c) 4 (d) -1

SOL

....

....

F the domain of the function f : $f(x) = \log_{(1-x)} x$ is ....

(a) $x > 0$ (b) $x < 1$ (c) $0 < x < 1$ (d) $0 \le x \le 1$

SOL

………

F the domain of the function ƒ : ƒ (x) = $\log_{(1-x)} 3$ is …………

(a)] -∞ , 0 [∪] 0 , 1 [(b)] -∞ , 1 [(c)] 1 , ∞ [(d)] -1 , 1 [

SOL

………

F if : log (x – 5) > 0 , then : …………

(a) $x > 5$ (b) $x > 1$ (c) $x > 6$ (d) $x < 5$

SOL

………

F if : $\log_3 5 = a$, then : $\log_{15} 5$ = ……………

(a) a^2 (b) $3a$ (c) $\frac{1}{a+3}$ (d) $\frac{a}{a+1}$

SOL

………

F ABC is a right-angled triangle at A in which :

AB = ($\log_4 3$) cm , AC = ($\log_3 64$) cm , then : its area = ………… cm^2

(a) 1.5 (b) 3 (c) log 16 (d) $\log_3 16$